

# Perturbative study of Yang-Mills theory in the infrared

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(Dated: April 4, 2016)

Pure Yang-Mills  $SU(N)$  theory is studied in four dimensional space and Landau gauge by a double perturbative expansion based on a *massive* free-particle propagator. By dimensional regularization, all diverging mass terms cancel exactly in the double expansion, without the need to include mass counterterms that would spoil the symmetry of the original Lagrangian. The emerging perturbation theory is safe in the infrared and shares the same behaviour of the standard perturbation theory in the UV. At one-loop, gluon and ghost propagators are found in excellent agreement with the data of lattice simulations and an infrared-safe running coupling is derived. A natural scale  $m = 0.6 - 0.7$  GeV is extracted from the data for  $N = 3$ .

PACS numbers: 12.38.Bx, 12.38.Lg, 12.38.Aw, 14.70.Dj

Our knowledge of QCD is still limited by the lack of a powerful analytical method for the study of the infrared range. It is widely believed that perturbation theory breaks down at the low-energy scale  $\Lambda_{QCD} \approx 200$  MeV and the study of very important phenomena, including hadronization and quark confinement, must rely on phenomenological models or numerical lattice simulations. Even pure Yang-Mills  $SU(N)$  theory is still not fully understood in its infrared limit. Important progresses have been achieved in the last years by developing new analytical tools[1–31] and by simulating larger and larger lattices[32–34].

A key role is played by the dynamical mass[35] that the gluon seems to acquire in the infrared according to almost all non-perturbative studies. Moreover, it has been shown that the inclusion of a mass by hand leads to a phenomenological model that can be studied by perturbation theory[36–38].

In a recent paper[22], we pointed out that a massive perturbation theory, based on a massive free-particle propagator can be developed by an unconventional setting of the perturbative method, without changing the Lagrangian and without adding free parameters that were not in the original Lagrangian, yielding a first-principle analytical treatment based on perturbation theory.

In this paper the idea is developed by a double expansion in dimensional regularization. Pure Yang-Mills  $SU(N)$  theory is studied in four dimensional space and Landau gauge by an expansion in powers of the total interaction and of the coupling. By dimensional regularization, all diverging mass terms cancel exactly in the double expansion, without the need to include mass counterterms that would spoil the symmetry of the original Lagrangian. The emerging perturbation theory is safe in the infrared and shares the same behaviour of the standard perturbation theory in the UV. While the present letter summarizes the main results, more details of the calculation will be published elsewhere[39].

Let us consider pure Yang-Mills  $SU(N)$  gauge theory without external fermions. The Lagrangian can be written as  $\mathcal{L} = \mathcal{L}_{YM} + \mathcal{L}_{fix}$  where  $\mathcal{L}_{YM}$  is the standard Yang-Mills term and  $\mathcal{L}_{fix}$  is the gauge fixing term. Usually the total Lagrangian is split into two parts, a free-particle Lagrangian  $\mathcal{L}_0$  that does not depend on the coupling strength  $g$ , and an interaction  $\mathcal{L}_{int}$  that contains  $\mathcal{O}(g)$  and  $\mathcal{O}(g^2)$  terms. In Landau gauge the gluon free-particle propagator is transverse

$$\Delta_0^{\mu\nu}(p) = t^{\mu\nu}(p) \frac{1}{-p^2}; \quad t^{\mu\nu}(p) = g^{\mu\nu} - \frac{p^\mu p^\nu}{p^2} \quad (1)$$

and has a pole at  $p^2 = 0$ . We can shift the pole and modify the free-particle propagator as

$$\Delta_0^{\mu\nu}(p) \rightarrow \Delta_m^{\mu\nu}(p) = t^{\mu\nu}(p) \frac{1}{-p^2 + m^2} \quad (2)$$

without changing the content of the theory provided that the counterterm

$$\delta\mathcal{L} = \frac{1}{2} m^2 A_\mu A^\mu \quad (3)$$

is added to the interaction. Actually, we are just adding and subtracting the same quantity in  $\mathcal{L}_0$  and  $\mathcal{L}_{int}$  without changing the total Lagrangian. Thus we can develop a perturbative expansion in powers of the total interaction and use the standard formalism of perturbation theory with a total interaction  $\mathcal{L}_{int} + \delta\mathcal{L}$  that is a mixture of terms that depend on the coupling strength  $g$  and the counterterm that does not vanish in the limit  $g \rightarrow 0$ . In the expansion, the free-particle propagator is the massive propagator (2), and the vertices are of order  $\mathcal{O}(g^0)$  (the counterterm), order  $\mathcal{O}(g)$  (the three-particle gluon and ghost-gluon vertex) and order  $\mathcal{O}(g^2)$  (the four-gluon vertex). That the content of the theory has not changed can be easily seen by summing up all graphs with  $n$  insertions of the counterterm in a gluon line. As shown in Fig. 1, we can write a dressed propagator as the infinite

$$\text{chain of circles} = \text{chain of circles} + \text{chain of circles with cross} + \text{chain of circles with two crosses} + \text{chain of circles with three crosses} + \dots$$

$$\text{chain of circles} = \Delta_m(p) \quad \text{cross} = m^2$$

Figure 1: Graphical illustration of Eq.(4). The cross is the counterterm of Eq.(3) that gives a factor  $m^2$ .

sum of a set of reducible graphs

$$\Delta(p) = \frac{1}{-p^2 + m^2} \sum_{n=0}^{\infty} \left[ m^2 \frac{1}{-p^2 + m^2} \right]^n = \frac{1}{-p^2}. \quad (4)$$

The same result can be described as the effect of a proper polarization term  $\Pi = m^2$  that arises from the counterterm and cancels the mass. Then formally, the two expansions are equivalent if we sum up all graphs. On the other hand, by the same token, at any finite order, the perturbation theory that we develop is not equivalent to the standard perturbation theory, but the two expansions differ by an infinite class of graphs that introduce non-perturbative effects. In other words, the two expansions differ by some non-perturbative content at any finite order. In fact, it is well known that the gauge invariance of the theory does not allow any shift of the pole in the propagator at any finite order, so that the massive zeroth order propagator  $\Delta_m$  cannot be obtained by the standard perturbation theory at any finite order.

Since we already know, by non-perturbative calculations, that the gluon propagator is massive in the infrared, then we expect that the present expansion, with a massive zeroth-order propagator, would be more reliable than the standard expansion, at least in the infrared.

While massive models have been studied before and found in fair agreement with the lattice data[36–38], the present approach is very different because the Lagrangian is not modified and does not break BRST symmetry, so that no free parameters are added to the exact Yang-Mills theory, yielding a description that is based on first principles and can be improved order by order. For instance, we can easily show that no mass is predicted for the photon by the same method.

Having left the Lagrangian unchanged, we expect that the massive expansion should share the same behaviour of the standard expansion in the UV where any finite mass becomes negligible. In fact, if  $p^2 \gg m^2$  the geometric expansion in Eq.(4) is convergent and the two perturbation theories must give the same results. On the other hand, when  $p^2 \rightarrow m^2$  each single term of the geometric expansion Eq.(4) diverges and the formal sum of infinite poles introduces some non-perturbative content[23]. Thus we can predict that the scale  $m$  should be close to the Landau pole  $\Lambda$  where the standard perturbation theory breaks down.

An other interesting aspect of the present massive expansion is that no other mass counterterms are required.

Thus there is no need to include terms that, because of gauge invariance, were not in the original Lagrangian. All diverging mass terms are cancelled exactly by the counterterm  $\delta\mathcal{L}$ . We expected that a cancellation like that would arise by summing all graphs, just because the Lagrangian has not been changed and no diverging mass terms are present in the standard expansion. However, if we inspect the graphs in Fig.2, we can easily see that any insertion of  $\delta\mathcal{L}$  in a loop reduces the degree of divergence of the graph, and all mass terms become finite by a finite number of insertions. Thus, if the divergences must cancel, they will cancel at a finite order of the expansion provided that we retain more counterterm insertions than loops.

As pointed out before, the order of a graph is the number of vertices that are included, while the number of loops is equal to the powers of  $g^2$  in the graph. If the effective coupling is small, as it turns out to be according to non-perturbative calculations, we could consider a double expansion in powers of the total interaction and in powers of the coupling: we can expand up to the order  $n$ , retaining graphs with  $n$  vertices at most, and then neglect all graphs with more than  $\ell$  loops. If  $n$  is large enough, then all divergences in the mass terms are cancelled by the counterterms in the loops. For instance, at one loop we only need  $n = 3$ .

In this paper we report the results for a third-order one-loop double expansion in dimensional regularization. The gluon polarization and the ghost self-energy are evaluated by the sum of all graphs with no more than three vertices and no more than one loop, as shown in Fig.2. The integrals are evaluated analytically by dimensional regularization and expanded in powers of  $\epsilon = d - 4$ .

In dimensional regularization, the cancellation of all diverging mass-terms can be easily proven by a simple argument. The insertion of just one counterterm in a

$$\Sigma = - \text{chain of circles with cross} + \text{chain of circles with cross}$$

$$\Pi = \text{(1a)} + \text{(1b)} + \text{(1c)} + \text{(1d)} + \text{(2a)} + \text{(2b)} + \text{(2c)}$$

Figure 2: Two-point graphs with no more than three vertices and no more than one loop. The ghost self energy and the gluon polarization contributing to the functions  $F$  and  $G$  are obtained by the sum of all the graphs in the figure.

loop can be seen as the replacement

$$\frac{1}{-p^2 + m^2} \rightarrow \frac{1}{-p^2 + m^2} m^2 \frac{1}{-p^2 + m^2} = -m^2 \frac{\partial}{\partial m^2} \Delta_m \quad (5)$$

in the internal gluon line. If the graph has no other counterterm insertions, then its dependence on  $m^2$  comes only from the massive propagators and a derivative of the whole  $n$ th-order  $\ell$ -loop graph gives the sum of all  $(n+1)$ -order  $\ell$ -loop graphs that can be written by a single insertion in any possible ways. According to Eq.(5), each diverging mass term  $m^2/\epsilon$  that comes from a loop would give a crossed-loop term  $-m^2/\epsilon$ . The argument also suggests a simple way to evaluate the crossed-loop graphs by Eq.(5).

The cancellation of all diverging mass terms without inclusion of any other counterterm is very important because there is no need to include free parameters that were not in the Lagrangian, while all other divergences can be dealt with by standard wave function renormalization.

It is instructive to inspect the constant graphs that contribute to the proper gluon polarization  $\Pi$  in Fig 2. At the lowest order ( $n = 1, \ell = 0$ ) the counterterm  $\delta\mathcal{L}$  just adds the constant term  $\Pi_{1a} = m^2$  that cancels the shift of the pole in the propagator. The tadpole  $\Pi_{1b}$  is

$$\Pi_{1b} = \frac{3}{4} \alpha m^2 \left( \frac{2}{\epsilon} + \log \frac{\mu^2}{m^2} + \frac{1}{6} \right) \quad (6)$$

where the effective coupling  $\alpha$  is given by

$$\alpha = \frac{3N}{4\pi} \alpha_s; \quad \alpha_s = \frac{g^2}{4\pi}. \quad (7)$$

The crossed tadpole  $\Pi_{1c}$  can be evaluated directly or by a derivative according to Eq.(5)

$$\Pi_{1c} = -m^2 \frac{\partial \Pi_{1b}}{\partial m^2} = -\frac{3}{4} \alpha m^2 \left( \frac{2}{\epsilon} + \log \frac{\mu^2}{m^2} - \frac{5}{6} \right). \quad (8)$$

The diverging terms already cancel in the sum  $\Pi_{1b} + \Pi_{1c}$ . In fact, the double-crossed tadpole  $\Pi_{1d}$  is finite and including its symmetry factor it reads

$$\Pi_{1d} = -\frac{3}{8} \alpha m^2 \quad (9)$$

so that the sum is

$$\Pi_{1b} + \Pi_{1c} + \Pi_{1d} = \frac{3}{8} \alpha m^2. \quad (10)$$

While the ghost loop vanishes in the limit  $p \rightarrow 0$ , a mass term can arise from the gluon loop that in the same limit is

$$\Pi_{2b}(0) = -\alpha m^2 \left( \frac{2}{\epsilon} + \log \frac{\mu^2}{m^2} + \text{const.} \right) \quad (11)$$

and adding the crossed loop with its symmetry factor

$$\Pi_{2b}(0) + \Pi_{2c}(0) = \left( 1 - m^2 \frac{\partial}{\partial m^2} \right) \Pi_{2b} = -\alpha m^2. \quad (12)$$

Thus the one-loop gluon propagator reads

$$\Delta(p)^{-1} = -p^2 + \frac{5}{8} \alpha m^2 - [\Pi(p) - \Pi(0)]. \quad (13)$$

We observe that a finite mass term has survived, and it is of order  $\alpha$ . Since it only arises from the gluon loops, no mass would survive in QED for the photon by the same method.

The calculation of the total one-loop polarization and of the ghost self energy is straightforward but tedious. For a massive theory, the one-loop sunrise graphs  $\Pi_{2a}(p)$ ,  $\Pi_{2b}(p)$  and the one-loop ghost self energy have been evaluated by several authors. The crossed loops follow by a mass derivative according to Eq.(5). In the minimal subtraction scheme, all the divergences are cancelled by the same wave function renormalization constants of the standard expansion. Namely we obtain

$$Z_A - 1 = \frac{13\alpha}{9\epsilon}; \quad Z_\omega - 1 = \frac{\alpha}{2\epsilon}. \quad (14)$$

The resulting propagators are finite, and the ghost and gluon dressing functions,  $\chi$  and  $J$  respectively, can be written in units of the scale  $m^2$ . Without taking any special subtraction point, the dressing functions can be recast as

$$J(s) = \frac{J(s_1)}{1 + \alpha J(s_1) [F(s) - F(s_1)]} \quad (15)$$

$$\chi(s) = \frac{\chi(s_2)}{1 + \alpha \chi(s_2) [G(s) - G(s_2)]} \quad (16)$$

where  $s = p^2/m^2$ . The integration points  $s_1, s_2$  are arbitrary as also are the normalization constants  $J(s_1), \chi(s_2)$ . The functions  $F(x), G(x)$  do not depend on any scale or parameter and are given by the following explicit expressions

$$\begin{aligned} F(x) &= \frac{5}{8x} + \frac{1}{72} [L_A + L_B + L_C + R_A + R_B + R_C] \\ G(x) &= \frac{1}{12} [L_G + R_G] \end{aligned} \quad (17)$$

where the logarithmic functions  $L_X$  are

$$\begin{aligned} L_A(x) &= \frac{3x^3 - 34x^2 - 28x - 24}{x} \times \\ &\quad \times \sqrt{\frac{4+x}{x}} \log \left( \frac{\sqrt{4+x} - \sqrt{x}}{\sqrt{4+x} + \sqrt{x}} \right) \\ L_B(x) &= \frac{2(1+x)^2}{x^3} (3x^3 - 20x^2 + 11x - 2) \log(1+x) \\ L_C(x) &= (2 - 3x^2) \log(x) \\ L_G(x) &= \frac{(1+x)^2(2x-1)}{x^2} \log(1+x) - 2x \log(x) \end{aligned} \quad (18)$$

and the rational parts  $R_X$  are

$$\begin{aligned} R_A(x) &= -\frac{4+x}{x}(x^2 - 20x + 12) \\ R_B(x) &= \frac{2(1+x)^2}{x^2}(x^2 - 10x + 1) \\ R_C(x) &= \frac{2}{x^2} + 2 - x^2 \\ R_G(x) &= \frac{1}{x} + 2. \end{aligned} \quad (19)$$

In the UV the functions  $F, G$  have the asymptotic behaviour

$$F(x) \approx \frac{17}{18} + \frac{13}{18} \log(x); \quad G(x) \approx \frac{1}{3} + \frac{1}{4} \log(x) \quad (20)$$

so that the standard UV behaviour is recovered for  $s, s_0 \gg 1$

$$\begin{aligned} J(s)^{-1} &= 1 + \frac{13}{18} \alpha \log(s/s_0) \\ \chi(s)^{-1} &= 1 + \frac{1}{4} \alpha \log(s/s_0) \end{aligned} \quad (21)$$

as we could predict from the wave function renormalization constants Eq.(14).

In the opposite limit  $x \rightarrow 0$  we find  $G(x) \rightarrow \text{const.}$  and  $F(x) \sim (1/x)$  so that  $\chi(0)$  is finite and  $J(s) \sim (s/\alpha)$ , yielding a finite gluon propagator  $\Delta(0) = 8/5\alpha m^2$  as predicted by Eq.(13).

Up to a renormalization factor, the dressing functions are invariant for a change of the bare coupling. That is more evident if we consider the rescaled functions  $(\alpha J)$  and  $(\alpha \chi)$  since then Eqs.(15),(16) lose any explicit dependence on  $\alpha$ . In fact, the physical content of the theory is inside the universal functions  $F, G$ . The actual value of  $m^2$  takes the role of a natural scale that fixes the physical units and can only be determined by a comparison with some physical quantity, as for lattice simulations. That is just a consequence of the lack of an energy scale in the Lagrangian. However, the present calculation is very predictive and, up to irrelevant constants, the inverse dressing functions are predicted to take the universal shape of the functions  $F$  and  $G$ : we can write Eqs.(15),(16) as

$$\begin{aligned} [\alpha p^2 \Delta(p)]^{-1} &= F(p^2/m^2) + \text{const.} \\ [\alpha \chi(p)]^{-1} &= G(p^2/m^2) + \text{const.} \end{aligned} \quad (22)$$

where the constants depend on normalization, bare coupling and subtraction points. Thus we expect that, up to a scaling factor, all lattice data can be put on top of the plots of  $F$  and  $G$  by an additive constant. In Fig.3 and Fig.4 the lattice data of Ref.[32] are shown together with the plots of the functions  $F$  and  $G$ . The function  $F$  has a pronounced minimum at  $x \approx 1.62$  that is  $p \approx 0.93$  GeV in the units of the lattice data, thus fixing the scale  $m = 0.73$  GeV that is used in the figures. We also show

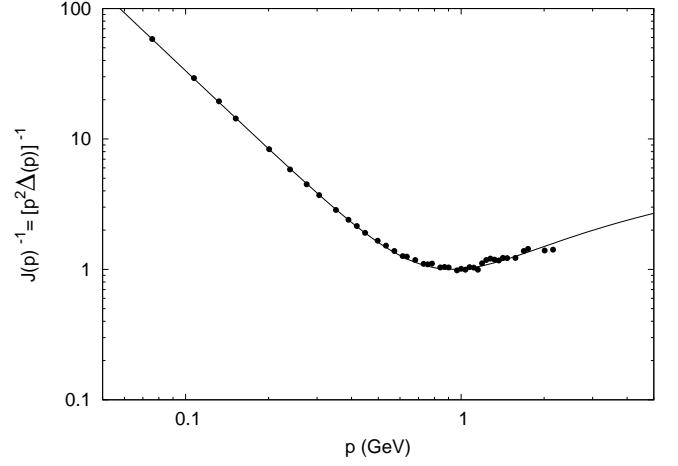


Figure 3: The function  $F(x) + c$  (line) is plotted together with the lattice data for the inverse gluon dressing function  $1/J(p) = p^2 \Delta(p)$  (points) extracted from the figure of Ref.[32] ( $N = 3$ ,  $g = 1.02$ ,  $L=96$ ) and scaled by a renormalization factor. The energy scale is set by taking  $m = 0.73$  GeV.

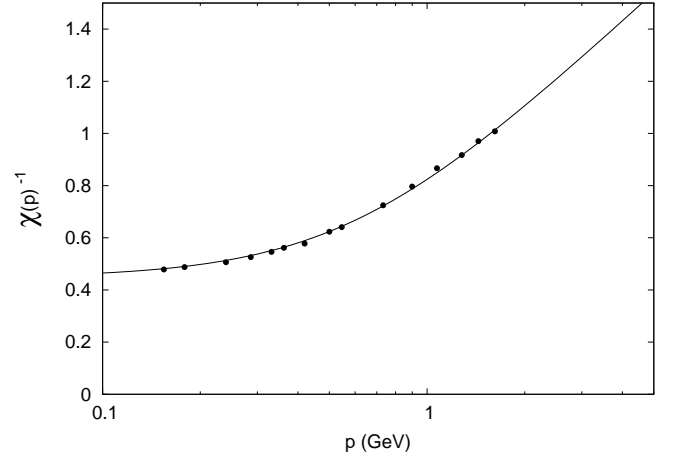


Figure 4: The function  $G(x) + c$  (line) is plotted together with the lattice data for the inverse ghost dressing function  $1/\chi(p)$  (points) extracted from the figure of Ref.[32] ( $N = 3$ ,  $g = 1.02$ ,  $L=96$ ) and scaled by a renormalization factor. The energy scale is set by taking  $m = 0.73$  GeV.

the gluon propagator and the ghost dressing function in Fig.5 and Fig.6.

The agreement is very good for a one-loop calculation but deviations can be expected when  $s$  is far from the subtraction point. Thus a slight dependence on the subtraction point would be a natural consequence of the one-loop approximation.

Nevertheless, let us pretend that we can ignore such limitations and look at observable quantities like the running coupling that can be related to phenomenology. Assuming that in Landau gauge the ghost-gluon vertex is regular[40] and the vertex renormalization constant can be set to one in a momentum-subtraction scheme,

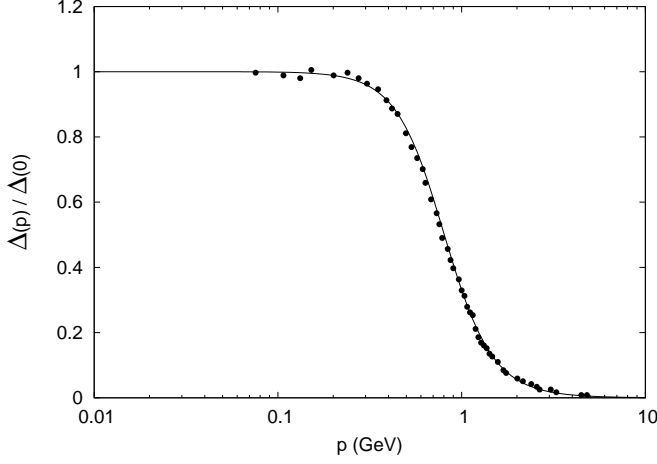


Figure 5: The gluon propagator  $\Delta(p)$  obtained by Eq.(15) (line) is plotted together with the lattice data (points) extracted from the figure of Ref.[32] ( $N = 3$ ,  $g = 1.02$ ,  $L=96$ ) and scaled by a renormalization factor. The energy scale is set by taking  $m = 0.73$  GeV.

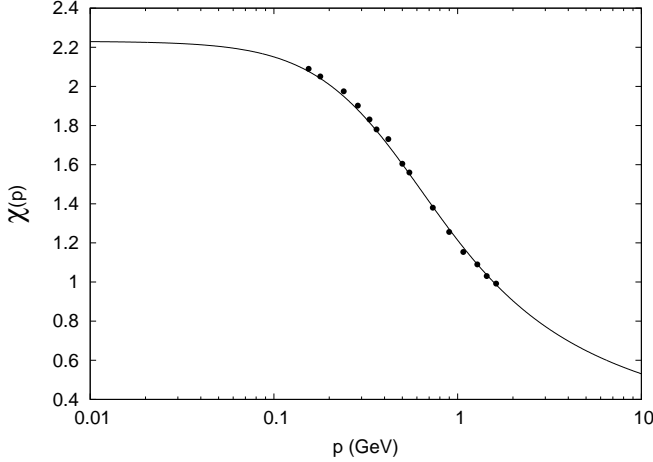


Figure 6: The ghost dressing function  $\chi(p)$  (line) is plotted together with the lattice data (points) extracted from the figure of Ref.[32] ( $N = 3$ ,  $g = 1.02$ ,  $L=96$ ) and scaled by a renormalization factor. The energy scale is set by taking  $m = 0.73$  GeV.

a running coupling is usually defined by the renormalization group invariant product  $\alpha(s) = \alpha_0 J(s) \chi(s)^2$  where  $J(s_0) = \chi(s_0) = 1$  and  $\alpha_0 = \alpha(s_0)$ . Using Eqs.(15) and (16), the one-loop running coupling can be written as

$$\alpha(s) = \frac{\alpha(s_0)}{1 + \alpha(s_0) [S(s) - S(s_0)]} \quad (23)$$

where  $S(x) = F(x) + 2G(x)$ .

By Eq.(20), for  $s, s_0 \gg 1$  we find the standard UV behaviour  $\alpha[S(s) - S(s_0)] \approx (11\alpha/9) \log(s/s_0) = (11N\alpha_s/12\pi) \log(s/s_0)$ . In this limit the result does not depend on the scale  $m$  and we recover the well known one-loop running coupling.

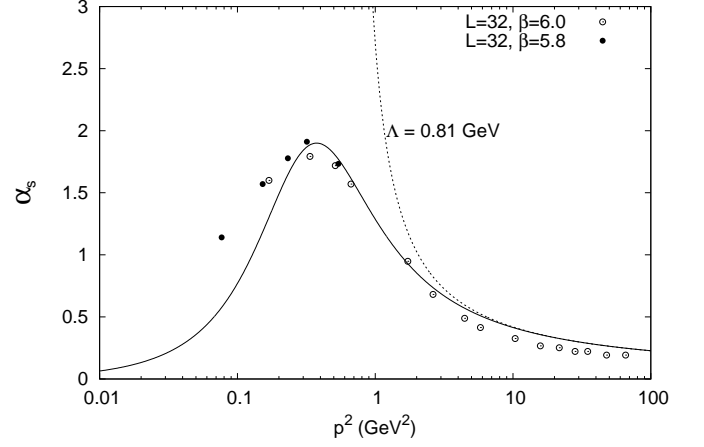


Figure 7: The running coupling  $\alpha_s = 4\pi\alpha/(3N)$  for  $N = 3$  by Eq.(23).  $s_0$  and  $\alpha_s(s_0)$  are fixed at the maximum ( $s_0 = x_M = 1.044$ ,  $\alpha_s(s_0) = 1.9$ ) and  $m = 0.6$  GeV (solid line). The points are the lattice data of Ref.[34]. The broken line is the standard one-loop behaviour for  $\Lambda = 0.81$  GeV.

In the infrared, as shown in Fig.7, the running coupling is finite, it does not encounter a Landau pole and vanishes in the limit  $s \rightarrow 0$  as the power  $\alpha(s) \sim s$ . A maximum is found at the point where  $dS(x)/dx = 0$ , which occurs at  $x_M = 1.044$ . This point does not depend on any parameter, and can be used as an alternative method for fixing a physical energy scale if that energy is *measured* somehow. For  $N = 3$ , many lattice simulations predict a maximum at  $p \approx 0.6 - 0.7$  GeV giving a scale  $m \approx 0.6$  GeV that is not too far from the value  $m = 0.73$  that was extracted from the propagators of Ref.[32]. Probably, this small difference would be narrowed by the inclusion of higher loops in the calculation.

While in the UV the asymptotic behaviour of the coupling seems to give no information on the scale  $m$ , in the infrared a phenomenological knowledge of the coupling would fix all the free parameters. Suppose that we pinpoint the value of  $\alpha_s$  at its maximum,  $\alpha_s \approx 1.9$  according to Ref.[34], and let us explore as an exercise the behaviour of the coupling when we go back towards the UV by Eq.(23), with all the limitations of the one-loop approximation. The maximum at  $p \approx 0.6$  gives a scale  $m \approx 0.6$  GeV. We can set  $s_0$  at this point  $s_0 = x_M$  and then take  $\alpha_s(s_0)$  as the maximum coupling. At the same point  $S(x_M) = S_M = 3.09$ . Then, pushing  $s$  towards the UV and making use of the asymptotic behaviour Eq.(20), we can insert  $S(s) \approx 29/18 + (11/9) \log(s)$  in Eq.(23) and write it as a standard one-loop coupling

$$\alpha_s(\mu^2) \approx \frac{12\pi}{11N \log(\mu^2/\Lambda^2)} \quad (24)$$

where the scale  $\Lambda$  is defined as

$$\Lambda = b m \exp \left[ -\frac{12\pi}{22N\alpha_s(x_M)} \right] \quad (25)$$

and the coefficient  $b$  satisfies  $b^2 = \exp(9S_M/11 - 29/22) = 3.35$ . Eq.(25) provides a direct link between the Landau pole of the standard one-loop running coupling at  $p = \Lambda$  and the infrared parameters, namely the mass scale  $m$  and the maximum value of the coupling  $\alpha_s(x_M)$ . Inserting the value  $\alpha_s(s_0) \approx 1.9$  of Ref.[34] in Eq.(25) we obtain  $\Lambda = 0.81$  GeV for  $N = 3$ , which is not too far from the value  $\Lambda = 0.7$  GeV that is used in the same paper for a fit of the lattice data in the UV. Moreover, we can extract from Eq.(25) the pure theoretical bound  $m > \Lambda/b \approx 0.55\Lambda$ .

Of course, the calculation is just a one-loop approximation. It generalizes the standard one-loop results to the infrared, but maintains the limitations of neglecting higher order terms. Moreover, when connecting very different scales, the use of renormalization group techniques becomes mandatory for a quantitative description and has been shown to be very effective in other massive models[36–38].

In summary, the double expansion has been shown to be viable for energies ranging from the UV to the infrared, without changing the original Lagrangian, reaching a good agreement with the lattice data from first principles. While the full potentialities of the method are totally unexplored yet, it might extend the standard perturbative approach to lower energies deep inside the *non-perturbative* sector of QCD.

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